We propose a simplified method for estimating the five elastic stiffnesses of unidirectional carbon fiber reinforced plastic (UD-CFRP) coupon by using the velocity anisotropy of laser excited Lamb waves. The method utilizes the in-plane orientation dependency of the Rayleigh wave velocities which are estimated from the velocity dispersions of the zero-th order anti-symmetric Lamb (Ao-Lamb) wave. We estimated five elastic stiffnesses of UD-CFRP coupons by utilizing the simplex-assisted inverse scheme, and compared with those determined by the strain gauge method. The proposed method was found to be effective for estimating the stiffnesses of anisotropic material. However, the inverse scheme must be modified to improve the accuracy of some stiffnesses.

KEYWORDS: elastic stiffness, simplex-assisted inversion, transverse isotropic, Lamb wave, Rayleigh wave, in-plane orientation dependency, UD-CFRP
1. Introduction

CFRP plate is reported to suffer various types of internal damages due to its anisotopic characteristics at being locally loaded. Computer simulation and monitoring of acoustic waves due to the internal damages are important engineering problems in transportation vehicles. Classification of such fracture modes as the delamination and transverse crack in CFRP is now becoming possible by the quantitative waveform simulation of the zero-th order Lamb waves when the elastic stiffnesses are given. In order to accomplish such waveform simulation effectively, a simplified estimation method of elastic stiffnesses of anisotropic material is urgently required. As the elastic properties of FRP generally change place to place, a simplified estimation method, rather the sophisticated time-consuming method for estimating the accurate properties, is desired.

Extensive researches of elastic stiffness estimation have been attempted so far. Some researches utilized the Rayleigh wave and some the Lamb waves. The elastic stiffnesses of anisotropic material were accurately estimated by using either of the phase and group velocities of surface acoustic waves (SAWs). For instance, Wu et.al\textsuperscript{1} estimated the elastic stiffnesses of UD-GFRP using the SAW energy velocities measured by point transmitter and a point receiving system (PT/PR system). The stiffnesses were successfully estimated by using the Stroh-Barnett’s integral formalism. Deschamps et.al\textsuperscript{2} proposed a numerical method for estimating the elastic properties from the group velocity distribution in the principal planes. They estimated the stiffnesses by employing the least square fit for the Cagniard-de Hoop contour. These methods can give accurate elastic stiffnesses when reliable orientation dependency of group velocities of SAW are measured by a PT/PR system, but requires enormous computation cost.

Park et.al\textsuperscript{3} first explored the potential for application of line-laser induced Rayleigh wave phase velocities. They excited the Rayleigh wave with a high directionality by using a line-focused short pulse YAG laser, and obtained the orientation dependency of the phase velocity of the Rayleigh wave. Five elastic stiffnesses of a UD-GFRP were estimated by using the eigen value analysis. The procedure for stiffness estimation was significantly simplified by an adoption of a line-focused laser SAW system.

Employment of transmitting quasi-bulk waves was also attempted by Castagnede et.al\textsuperscript{4}. They determined the elastic constants of UD-GFRP by utilizing the laser generated transmitting waves. The near phase velocities of
quasi-longitudinal, pure and quasi-shear waves in principal planes were monitored. The wave speed determined by travelling time of pin-point laser ultrasonic is reported to be sufficiently close to the phase velocities, which permit the utilization of simplified inverse scheme.

Kawashima et al. measured the phase velocities of quasi-bulk waves through a ceramic fiber reinforced metal composite by using the double through-transmission ultrasonic method in water. Usage of liquid couplant inevitably limits the application to large structural components.

For a single crystal, the orientation dependency of the laser-excited Rayleigh wave velocities has been successfully utilized for accurate stiffness estimation. These researches can be found elsewhere. Chubachi et al. utilized the line-focus acoustic microscope which allows the measurement of SAW velocity in a single prescribed direction. This method has been widely used to measure the anisotropic dependence of SAW velocity on the propagation direction.

Contrary to single crystals, FRPs show local inhomogeneity due to the scattering of fiber content, and therefore the elastic stiffnesses generally variate place to place. A simple and quick estimation method, which inevitably gives approximate elastic properties, is acceptable for FRPs. The accuracy of the elastic stiffness for waveform simulation is generally enough in the two order of significant figures.

Attempted is to estimate the elastic stiffnesses of thin UD-CFRP coupon using the laser generated SAW. As the UD-CFRP plate is generally thin, the Lamb waves are monitored. Accurate estimation of elastic stiffness from the Lamb waves requires the determination of the velocity dispersions of higher order Lamb modes which are detectable by the oblique isonification apparatus (leaky guided waves in a water loaded specimen). Mal et. al. estimated the stiffnesses of UD-CFRP from the velocity dispersions of higher order symmetric and anti-symmetric Lamb modes. Water loading appears to be a problem.

Therefore, we propose a simplified in-air method to estimate the five elastic stiffnesses of the UD-CFRP plate by using the velocity anisotropy (in-plane orientation dependency) of Lamb waves. The Lamb waves were excited by a line-focused Q-switched YAG laser and monitored by a point-focused hetero-dyne type laser interferometer. We quantified the velocity dispersion of the Lamb waves by the continuous wavelet transform. The Rayleigh wave velocities, estimated from the velocity dispersion of the zero-th order anti-symmetric Lamb waves, were submitted to the simplex-assisted inverse scheme. As the
The proposed method involves approximations and restrictions in inverse processing, it is not applicable to general anisotropic materials. However, we can easily estimate the elastic stiffnesses of FRPs without using the sophisticated computer scheme. The elastic stiffnesses estimated are compared with the experimentally measured engineering Young’s modulus and Poisson’s ratio.

2. Experimental

Specimen is the UD-CFRP plate made of 24 layer pre-prig sheets of 0.091 mm thickness. The content of carbon fiber and density are 70%wt. and 1776kg/m³, respectively. The specimen employed for laser SAW test has dimension of 47 mm length, 30 mm width and 0.872 mm thickness.

Figure 1 shows the relationship between the fiber direction and wave propagation. Setting the principle axis of symmetry (fiber direction) as the direction of X₁, the plane X₂-X₃ is isotropic. The elastic stiffnesses of transversely isotropic FRP are given by the matrix form of Eq.(1)

$$
\begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
$$

(1)

Figure 2 shows schematic illustration of laser Lamb system. We excited the Lamb waves by adiabatic thermal expansion of a line focused Q-switched YAG laser (half value duration time of 5ns) of 0.04 mm width and 4 mm length. They were excited in the direction normal to the laser line, and monitored by a point-focused hetero-dyne type laser interferometer at distance of 6.54mm. The wave propagation distance was accurately controlled within 2 μm by using the computer control stepping motor. The specimen was rotated with 15 degree increment by computer controlled in-plane rotating stage. The output of the interferometer was digitized by a fast A/D converter (digitizer) at sampling interval of 20ns. The A/D converter was triggered accurately by a photodiode. This optical triggering method can eliminate most significant triggering time error except for the photodiode rise time and light flight time. The velocity dispersions of the Lamb waves were obtained by the continuous wavelet transform with Gabor function as the mother wavelet. Detail processing of wavelet transform can be found elsewhere¹¹,¹².
3. Inversion Processing for Estimating the Elastic Stiffnesses

The proposed processing is based on the assumption that the elastic stiffnesses of transverse isotropic material can be estimated from the in-plane orientation dependency of the Rayleigh wave velocities which are estimated from the Lamb waves. As being expressed by Eq.(2), the Rayleigh wave velocity \( V_r \) of an isotropic elastic and homogenous material is given as a function of the phase velocities of the longitudinal \( V_l \) and shear waves \( V_t \).

\[
\left( \frac{V_r}{V_l} \right)^b - 8 \left( \frac{V_r}{V_t} \right) + 8 \left( 3 - 2 \frac{V_r}{V_t} \right) \left( \frac{V_r}{V_t} \right)^2 - 16 \left( 1 - \left( \frac{V_r}{V_t} \right)^2 \right) = 0 \quad (2)
\]

The Rayleigh velocity of anisotropic medium \(^3\) and the velocity dispersion of laminated anisotropic medium \(^13\) were calculated by the Christoffel’s equation and the matrix transfer method, respectively. We once attempted to estimate the stiffnesses of unidirectional glass fiber reinforced plastic by the inverse processing of the Rayleigh wave velocities based on the Christoffel’s equation, however it took enormous computation cost. Then we employed a simplified method in which Eq.(2) can be extended to a transverse isotropic material. According to Castagnede et al \(^4\), the velocity of pure shear \( V_t \) and longitudinal wave \( V_l \) are given by Eq.(3) and (4), respectively.

\[
V_t = \sqrt{\frac{C_{44} \sin^2 \theta + C_{55} \cos^2 \theta}{\rho}} \quad (3)
\]

\[
V_l = \sqrt{\frac{a + b}{2\rho}} \quad (4)
\]

where \( a \) and \( b \) are given by Eqs.(5) and (6), respectively. \( \rho \) is the density of medium. \( \theta \) designates the angle between the wave propagation direction and fiber direction in the principal planes.

\[
a \equiv C_{22} \sin^2 \theta + C_{11} \cos^2 \theta + C_{55} \quad (5)
\]

\[
b \equiv \left[ \left( C_{22} - C_{55} \right) \sin^2 \theta + \left( C_{55} - C_{11} \right) \cos^2 \theta \right]^{\frac{1}{2}} + \left( C_{11} + C_{66} \right) \sin^2 2\theta \quad (6)
\]

This implies that the elastic stiffnesses can be estimated from the in-plane orientation dependency of the \( V_l \) and \( V_t \) which are involved in the Rayleigh equation. In order to examine the propriety of this assumption, we first computed the orientation dependency of the Rayleigh wave velocity \( V_r(\theta) \) by Eq.(2), and compared with that computed by Christoffel’s equation (7).
\[
\det\left[C_{ijkl}n_in_j - \rho V^2 \delta_{ik}\right] = 0 \quad (7)
\]

Here \(C_{ijkl}\) are the elastic constants tensor and delta \(\delta_{ik}\) the Kronecker’s delta. The direction cosine of wave propagation are specified by \(n_j\) and \(n_i\).

Table 1 shows the elastic stiffnesses used for the trial computation. These are, as will be shown later, the stiffnesses of UD-CFRP estimated by the proposed method. Fig. 3 compares the orientation dependency of the Rayleigh velocities in the principal plane \(X_1\cdot X_2\). The velocity distribution computed by the proposed method (solid line) agrees well with that by the Christoffel’s equation (dotted line) except \(\theta < 30^\circ\). The velocities near the fiber direction (\(\theta = 0^\circ\)) by the proposed method is 4% smaller than that by the Christoffel’s equation. However, disagreement at this level is acceptable for our purpose. Indeed, as will be discussed later, the UD-CFRP shows significant local fluctuation of Young’s modulus and Poisson’s ratio. Fluctuation of the Young’s modulus \(E_1\) in the fiber direction, determined by the strain gauge method, is found to reach 4.6%.

For the thin UD-CFRP plate, only the Lamb waves can be measured. Therefore the Rayleigh velocity must be determined from the velocity dispersion of the zero-th order Lamb modes. Fig. 4 compares the velocity dispersions of 0-th order anti-symmetric Lamb mode (Ao-Lamb) in the direction \(X_1\) (\(\theta = 0^\circ\)) and direction \(X_2\) (\(\theta = 90^\circ\)). The velocity dispersions are calculated by Eq.(8) and Christoffel’s equation by using the stiffnesses in Table 1.

\[
4LS \tanh\left(\frac{\pi fhS}{V}\right) - (1 + S^2) \tanh\left(\frac{\pi fhL}{V}\right) = 0 \quad (8)
\]

Here \(L\) and \(f\) denotes the thickness of plate and frequency, respectively. \(S\) and \(L\) are given by Eqs.(9) and (10), respectively.

\[
S = \sqrt{1 - \left(\frac{V}{V'}\right)^2} \quad (9)
\]

\[
L = \sqrt{1 - \left(\frac{V}{V'}\right)^2} \quad (10)
\]

The phase velocity of the Ao-Lamb increases with frequency, and reaches the Rayleigh velocity at higher frequency. The velocity dispersions in direction 2, calculated by Eqs (7) and (8), completely coincide. However, in the lower frequency range in direction \(X_1\), relatively large difference was observed. This
implies that if we use the velocity at higher frequency, the error can be minimized. A broad-band laser ultrasonic system enables us to determine the Rayleigh wave velocity at frequency of 7 MHz.

We developed the inversion scheme for estimating the five elastic stiffnesses from the in-plane distribution of Rayleigh velocities $V_r(\theta)$. Fig.5 shows the flow chart in the simplex-assisted inverse scheme. This scheme has four restrictions (indicated as restrictions I to IV) to improve the accuracy of the estimated stiffnesses. We first assume five stiffnesses, then examine whether these satisfy the following three restrictions.

Restriction I: anisotropy index $\eta = \frac{2C_{44}}{C_{11} - C_{12}}$ be less than 1.0
Restriction II: Poisson’s ratio be from 0 to 0.5
Restriction III: Poisson’s ratio $\nu_{12}$, calculated by the Betch’s reciprocal theorem, be less than $0.5\left(\frac{E_3}{E_1}\right)$

Data survey of the published paper4,10, 14, 15 on the fiber reinforced composites demonstrated the above three restrictions were reasonable for UD-FRP. The restriction IV, the surface skimming compressive wave (SSCW) along the fiber direction be in the range from 95% to 110% of the longitudinal wave velocity, was also used when SSCW is measured. However, we could not use it this time, because the SSCW was not measured for UD-CFRP.

Using the stiffnesses which satisfied the restrictions I to III, we then computed the orientation dependency of the Rayleigh velocities $V_r^*(\theta)$, and compared with the measured velocities $V_r(\theta)$. The summation of the velocity difference between two, i.e., $\sum |V_r^*(\theta) - V_r(\theta)|$, was used as a converging condition. If the conversion condition is not satisfied, new $V_r^*(\theta)$ were computed using the new stiffnesses estimated by the simplex method. Iteration was performed until the velocity difference satisfy the given value (45 m/s in this research).

4. Results and Discussion

Fig.6 show typical Lamb waves monitored in the directions $\theta = 0^\circ$, $45^\circ$ and $90^\circ$. We obtained the group velocity dispersion of these waves by using the continuous wavelet transform. Fig.7 show examples of bird’s eye view of the wavelet coefficients for the wave at $\theta = 0^\circ$ and $90^\circ$. The velocity dispersions were then calculated by dividing the propagation distance (6.54mm) with the maximum arrival time at each frequency. Shown in Fig.8 is the velocity dispersion of the Ao-Lamb wave. Data scatters at lower frequency range probably due to the short time sampling interval. However this is not so serious for estimating the Rayleigh velocity at high frequency. As the group velocity of
the Ao Lamb reaches the phase velocity of the Rayleigh wave at higher frequency, it was determined as the velocity at frequency of around 7MHz. Fig.9 shows orientation dependence of the Rayleigh velocities as a function of $\theta$. The broken line represents the velocity profile calculated by Eq.(2) using the stiffnesses in Table 1.

The change of elastic stiffnesses and error during the inverse processing is shown in Fig.10. Inadequate initial stiffnesses ($C_{11}=50$, $C_{22}=50$, $C_{12}=10$, $C_{44}=10$, $C_{55}=10$ GPa) resulted in abnormal stiffnesses after 276 iteration (first iteration) while the error itself was within 43 m/s. This is due to the trapping of convergence condition in a local minimum. Then the second iteration was attempted by using the new $C_{11}$ and $C_{22}$, given by the simplex scheme, while another stiffnesses are kept as the previous values. Five $C_{ij}$ after 224 iteration are shown in Table 1.

Fig.11 compares the experimental velocity dispersion of the Ao-Lamb wave, as a function of thickness $h \times$ frequency $f$, with those calculated by proposed method and Christoffel's equation. Contrary to the fairly good agreement between the three in direction X$_2$, deviation of experimental data from Christoffel's equation are apparent in the lower frequency range in direction X$_1$. It is noted that the dispersion curve by the proposed method agrees well with that by Christoffel's equation at $hf > 1.5$ MHzmm.

In order to examine the propriety of the estimated stiffnesses, we next measured the engineering Young's modulus and Poisson's ratio by the strain gauge method. Two specimens of 26 mm width and 120 mm length with fiber direction parallel or perpendicular to the loading axis were subjected to tensile tests. Two directional strain gauges with gauge length of 2 mm and width 1 mm were adhered at four different points for each specimen. The output of both the load cell and strain gauge were simultaneously monitored through a data logging system. Fig.12 compares stress-strain curves of the specimen with fiber direction parallel to loading. The curves (2) corresponds to the place which showed the lowest Young's modulus (146 GPa) while the curve (1) the highest modulus (160 GPa). The poisson’s ratio variates from 0.17 to 0.32. The large variation of the Poisson's ratio appears to be due to the local fluctuation of the carbon fibers as shown in the transverse microphotograph (Fig. 13). As the UD-GFRP is produced by plying of 24 pre-prig sheets, local inhomogeneity is apparent. Such inhomogeneity inevitably results in a large fluctuation of the Poisson’s ratio. Table 2 compares the estimated Young's moduli and Poisson's ratio (the upper row) with those experimentally determined ( the bottom row). The Poisson's
ratios estimated by Eqs.(11) and (12) are smaller than the experimental minimum values.

\[
\nu_{12} = \frac{C_{12}(C_{22} - C_{33})}{(C_{11}C_{22} - C_{12}^2)} \quad (11)
\]

\[
\nu_{21} = \frac{C_{12}}{(C_{22} + C_{33})} \quad (12)
\]

Equations suggest that the estimation error of \(C_{12}\) resulted in smaller Poisson’s ratio. The stiffness \(C_{12}\) appears to be estimated to the lower value. Accurate estimation of \(C_{12}\) is generally very difficult even by the another types ultrasonic method. However, estimation error of \(C_{12}\) can be improved by modifying the restriction in the inverse scheme, by setting a narrow range for anisotropic index, or by an adoption of additional restriction. This modification is left as the future problem.

5. Conclusion

We estimated the elastic stiffnesses of UD-CFRP by using both the Laser SAW system and the inversion scheme of the orientation dependent Rayleigh velocities.

Results are summarized below:

1) We utilized a laser SAW system in which the Lamb waves were excited by a line-focused Q-switched YAG laser and monitored by a heterodyne-type laser interferometer. The velocity of the Rayleigh wave were estimated from the group velocity dispersions of the zero-th order anti-symmetric Lamb wave (Ao-Lamb) obtained by the wavelet transform. Broad band laser SAW system enables one to estimate the in-plane orientation dependency of the phase velocity of the Rayleigh wave.

2) A simplified simplex-assisted inversion scheme for estimating the five elastic stiffnesses from the Rayleigh wave velocity anisotropy was proposed. Propriety and accuracy of the scheme was examined by the computation simulation of the velocity anisotropy of the Rayleigh wave. The error in estimating the stiffnesses was found to be allowable level for laminated UD-CFRP with local fluctuation of properties.

3) We estimated the five elastic stiffnesses and compared them with the engineering Young’s modulus and Poisson’s ratio measured by the strain gauge method. Two Young’s moduli \(E_1\) and \(E_2\) were found to agree well with the experimentally determined ones. However, the estimation of the Poisson’s ratio,
$v_{12}$ and $v_{21}$, appears to be poor due to the estimation error of $C_{12}$. Modification of the inverse scheme is needed.
Fig.1  Schematic Illustration of fiber direction and Laser SAW propagation direction in principal plane 
Fig.2  Laser SAW system for measuring the Lamb waves in UD-CFRP 
Fig.3  Comparison of the computed orientation dependency of the Rayleigh wave velocities in principal plane 
Fig.4  Computed velocity dispersions of the Lamb waves in direction 1 and 2. 
Fig.5 Flow chart for the Inverse processing of the Rayleigh wave velocities for estimating the elastic stiffnesses 
Fig. 6  Examples of detected Lamb waves for UD-CFRP 
Fig. 7  Bird’s eye view of the wavelet coefficients of the Ao-Lamb in direction $\theta$ =0° and 90° 
Fig. 8  Velocity dispersions of the Ao-Lamb 
Fig. 9  Orientation dependency of the Rayleigh wave velocities 
Fig. 10 Change of elastic stiffnesses (a) and error during iteration 
Fig.11 Comparison of the velocity dispersions computed by christoffel’s equation and simplified method with experimental data 
Fig. 12  Tensile stress- strain diagrams of the specimen with fiber direction parallel to loading. 
Fig. 13 Transverse microphotograph of laminated UD-CFRP of pre-prig sheets.
References
Table 1 Elastic stiffness of UD-GFRP estimated by the proposed method and used for computer simulation.

<table>
<thead>
<tr>
<th>Unit: GPa</th>
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<tbody>
<tr>
<td>$C_{11}$</td>
</tr>
<tr>
<td>168</td>
</tr>
</tbody>
</table>

Yoshihiro Mizutani

Table 2 Comparison of Young’s module and Poisson’s
ratio estimated by the strain gauge (a) and Rayleigh wave velocities (b)

<table>
<thead>
<tr>
<th></th>
<th>$E_1$(GPa)</th>
<th>$E_2$(GPa)</th>
<th>$\nu_{12}$</th>
<th>$\nu_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>146~160</td>
<td>6.2~6.8</td>
<td>0.02~0.032</td>
<td>0.17~0.32</td>
</tr>
<tr>
<td>(b)</td>
<td>168</td>
<td>6.3</td>
<td>0.005</td>
<td>0.121</td>
</tr>
</tbody>
</table>

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Fig. 1
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Fig. 2
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Fig. 3
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Fig. 4
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Fig. 5
Yoshihiro Mizutani
Fig. 6
Yoshihiro Mizutani
Fig. 7
Yoshihiro Mizutani
Fig. 8
Yoshihiro Mizutani

縦 3.9cm
横 8.6cm
Fig. 9
Yoshihiro Mizutani
Fig. 10
Yoshihiro Mizutani
Elastic stiffness, $C_{22}$, $C_{33}$, $C_{44}$, $C_{12}$ (GPa)

- $C_{11} = 168$
- $C_{44} = 2.1$
- $C_{55} = 5.5$

Fig. 11
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縦 10.3cm
横 8.6cm
Fig. 12
Yoshihiro Mizutani
Yoshihiro Mizutani
縦 6.7cm
横 8.6cm